Chapter 5:The IS-LM-Model

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Chapter 5:The IS-LM-Model

- Learning objectives

1. Derivation of the IS-curve
   - Slope of the IS curve
   - Shift of the IS-curve
2. Derivation of the LM-curve
   - Slope of the LM-curve
   - Shift of the LM-curve
3. Interplay of IS and LM equation
   - Graphic analysis
   - Calculation of the equilibrium values
   - Cramer's rule
4. Expansionary fiscal policy
   - Transmission
   - Income multiplier
   - Interest rate multiplier
5. Expansionary monetary policy
   - Transmission
   - Income multiplier
   - Interest rate multiplier
6. Fact check

Learning objectives chapter 5

After you worked through this chapter, you should know

a) which parameters influence the slope of the IS- and LM-curve and which changes lead to a shift of the IS- and LM-curve,

b) in which direction the curves shift *horizontally* and *vertically* in case that a shift parameter changes,

c) the income and interest rate effects of a monetary or fiscal policy and should know the uncertainties in the adjustment process,

d) how to describe the comparative static effects in a verbal graphical and formal way,

e) the dynamic adjustment process,

f) how you can use the matrix operations (Cramer‘s rule) to determine the income and interest rate levels in equilibrium as well as the multipliers.

Goods market and the IS equation

Consumption:	$C = c_0 + c_1 \cdot (Y - T)$
Investment:	$I = b_0 + b_1 \cdot Y - b_2 \cdot i$
Government spending:	$G = \bar{G}$
Aggregated demand:	$Z = C + I + G$
Equilibrium condition:	$Y = Z$

Goods market equilibrium condition

(1) $$Y = c_0 + c_1(Y - T) + b_0 + b_1 Y - b_2 i + G$$

(2) $$Y - c_1 Y - b_1 Y = c_0 - c_1 T + b_0 + G - b_2 i$$

(3) $$(1 - c_1 - b_1) \cdot Y = [c_0 - c_1 T + b_0 + G - b_2 i]$$

(4) $$Y = \frac{1}{1 - c_1 - b_1} [c_0 - c_1 T + b_0 + G - b_2 i]$$

Good market equilibrium condition at different interest rate levels

$$Y = \frac{1}{1 - c_1 - b_1} \cdot [c_0 - c_1 T + b_0 + G - b_2 i]$$

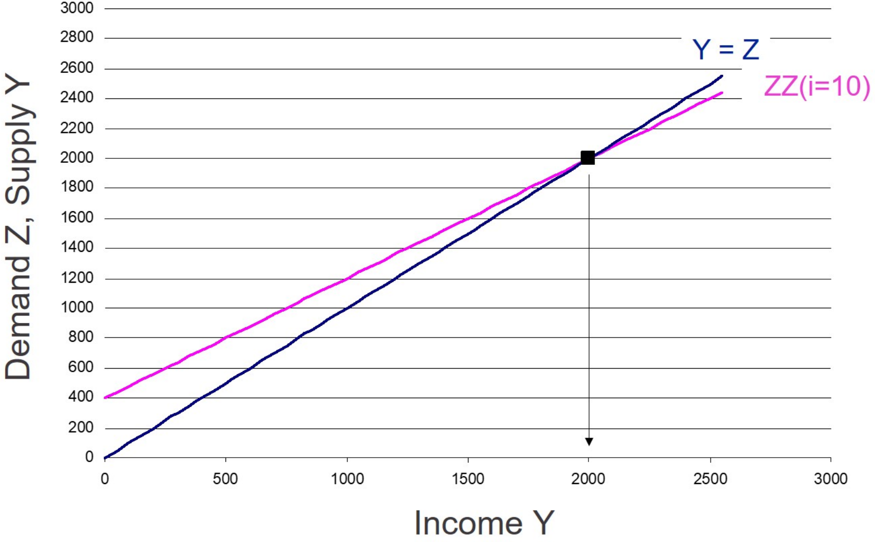
$c_1 = 0.75$
$b_1 = 0.05$
$b_2 = 20$
$c_0 = 400$
$b_0 = 150$
$T = 200$
$G = 200$

(5) $$Y_{i=10} = \frac{1}{1 - 0.75 - 0.05} \cdot [400 - 150 + 150 + 200 - 20 \cdot i] = \frac{1}{0.2} \cdot [600 - 20 \cdot 10] = 5 \cdot [400] = 2000$$

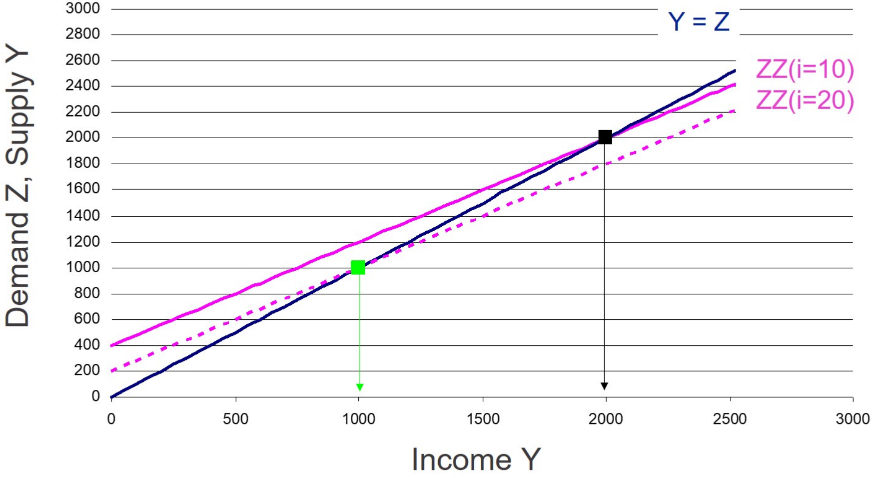
(6) $$Y_{i=20} = \frac{1}{0.2} \cdot [600 - 20 \cdot 20] = 5 \cdot [200] = 1000$$

(7) $$Y_{i=5} = \frac{1}{0.2} \cdot [600 - 20 \cdot 5] = 5 \cdot [500] = 2500$$

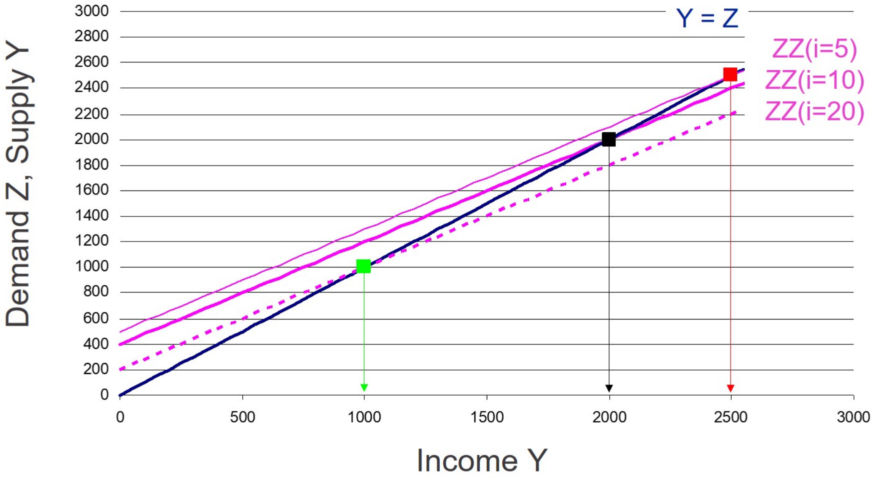
Goods market equilibrium at different interest rate levels



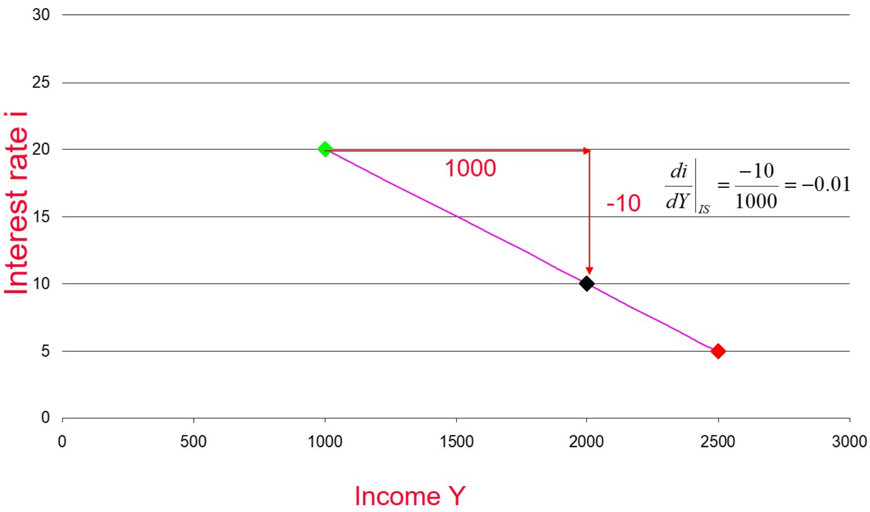
Goods market equilibrium at different interest rate levels



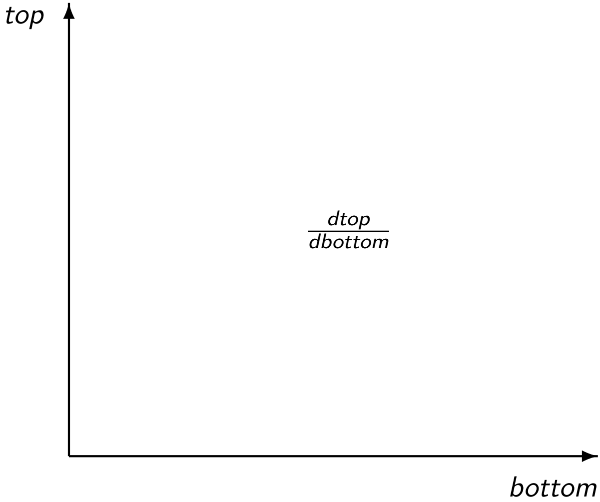
Goods market equilibrium at different interest rate levels



IS-curve in an interest rate-income-diagram



Slope of the IS-curve : $\frac{dtop}{dbottom}$



Slope of the IS-curve in an interest rate-income-diagram

$$(8) \qquad Y = \frac{1}{1 - c_1 - b_1} \cdot [c_0 - c_1 T + b_0 + G - b_2 i]$$

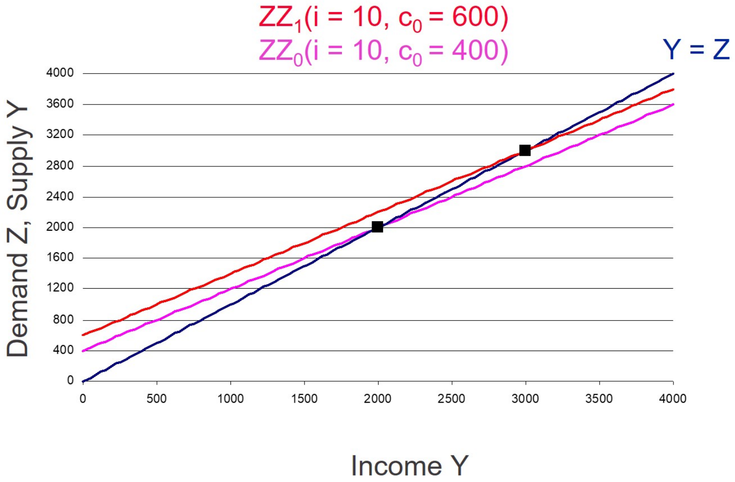
$$(9) \qquad dY = \frac{1}{1 - c_1 - b_1} \cdot [dc_0 - c_1 dT + db_0 + dG - b_2 di]$$

$$(10) \qquad dY = \frac{-b_2}{1 - c_1 - b_1} \cdot di$$

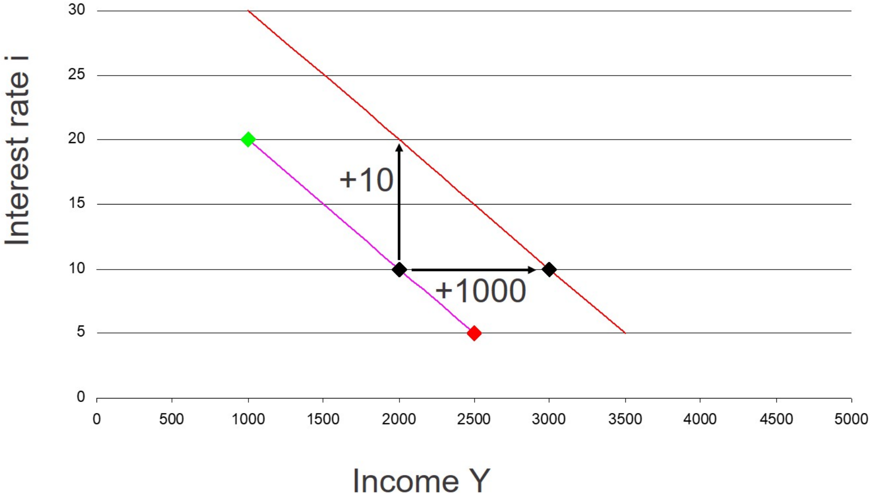
$$(11) \qquad \left.\frac{di}{dY}\right|_{IS} = -\frac{1 - c_1 - b_1}{b_2} < 0 \quad if \quad c_1 + b_1 < 1$$

$$(12) \qquad \left.\frac{di}{dY}\right|_{IS} = -\frac{1 - c_1 - b_1}{b_2} = -\frac{1 - 0.75 - 0.05}{20} = -\frac{0.2}{20} = -0.01$$

Shift of the IS-curve: Increase of the autonomous component of consumption (c_0)



Shift of the IS-curve



Horizontal shift of the IS-curve ($dc_0 = +200$)

(13) $$dY = \frac{1}{1 - c_1 - b_1}\left[dc_0 - c_1 dT + db_0 + dG - b_2 di\right]$$

(14) $$\frac{dY}{dc_0} = \frac{1}{1 - c_1 - b_1} > 0 \qquad \frac{dY}{dc_0} = \frac{1}{1 - 0.75 - 0.05} = 5$$

- $> 0 \Rightarrow$ Shift to the right
- Shift stronger, the larger c_1 and b_1
- Here: $dc_0 = 200 \Rightarrow dY = 5 \cdot 200 = 1000$

Vertical shift of the IS-curve ($dc_0 = +200$)

(15) $$dY = \frac{1}{1 - c_1 - b_1}\left[dc_0 - c_1 dT + db_0 + dG - b_2 di\right]$$

(16) $$0 = \frac{1}{1 - c_1 - b_1} dc_0 - \frac{b_2}{1 - c_1 - b_1} di$$

(17) $$\frac{b_2}{1 - c_1 - b_1} di = \frac{1}{1 - c_1 - b_1} dc_0$$

(18) $$\frac{di}{dc_0} = \frac{1}{b_2} > 0$$

Vertical shift of the IS-curve ($dc_0 = +200$)

$$\frac{di}{dc_0} = \frac{1}{b_2} > 0$$

(19)
$$\frac{di}{dc_0} = \frac{1}{20} = 0.05$$

- $> 0 \Rightarrow$ Curve shift upwards
- Shift stronger, the smaller b_2
- Here: $dc_0 = 200 \Rightarrow di = 0.05 \cdot 200 = 10$

Shift of an equilibrium curve

An equilibrium curve shifts,

- if a variable changes,
- which is contained in the equilibrium condition,
- but is not indicated on either of the two axes.

- The IS-curve shifts to the right if a variable changes, which has a positive impact on the demand for goods.
- $c_0 \uparrow \quad T \downarrow \quad b_0 \uparrow \quad G \uparrow$

Real money supply and real money demand

Real money demand: $M^D = d_0 + d_1 Y - d_2 i$

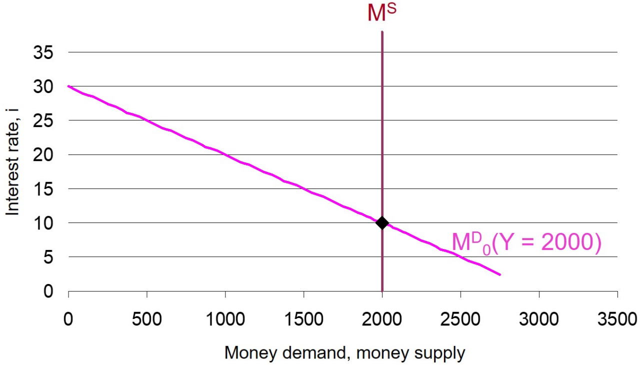
Real money supply: $M^S = \frac{M}{P}$

Equilibrium: $\frac{M}{P} = d_0 + d_1 Y - d_2 i$

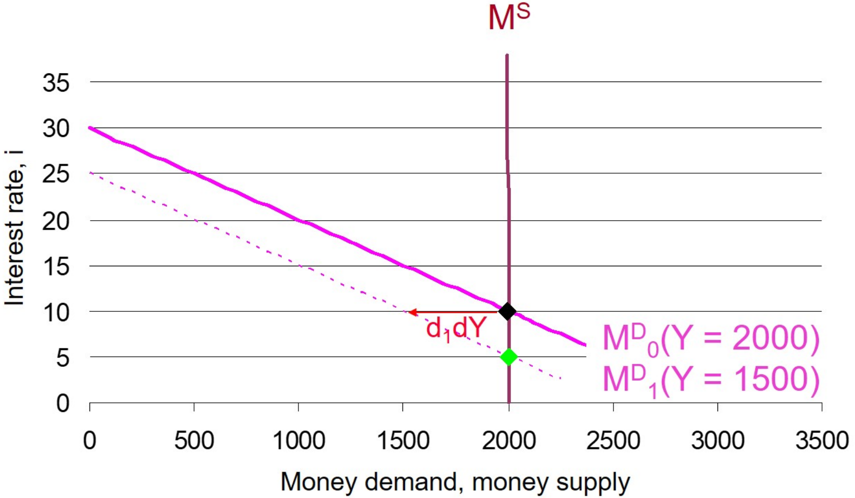
Money market equilibrium and the LM-curve, $(Y = 2000)$

$$\frac{M}{P} = d_0 + d_1 Y - d_2 i$$

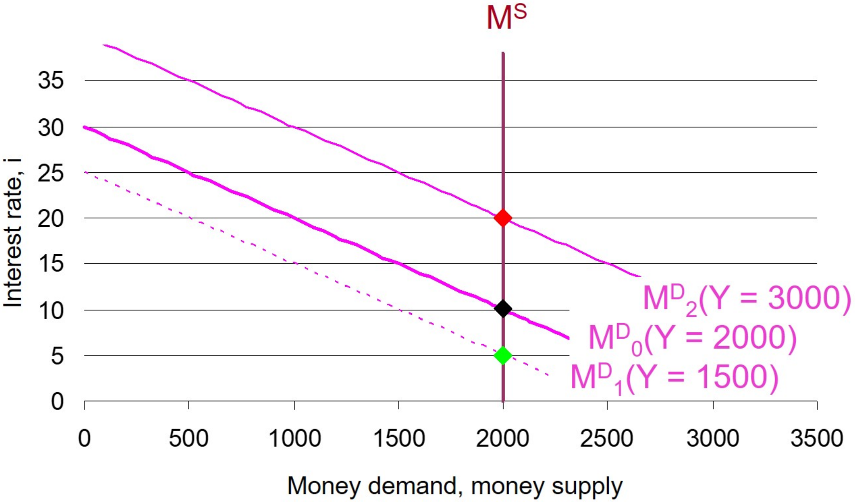
d_1	$= 1$
d_2	$= 100$
d_0	$= 1000$
Y	$= 2000$
M	$= 2000$
P	$= 1$



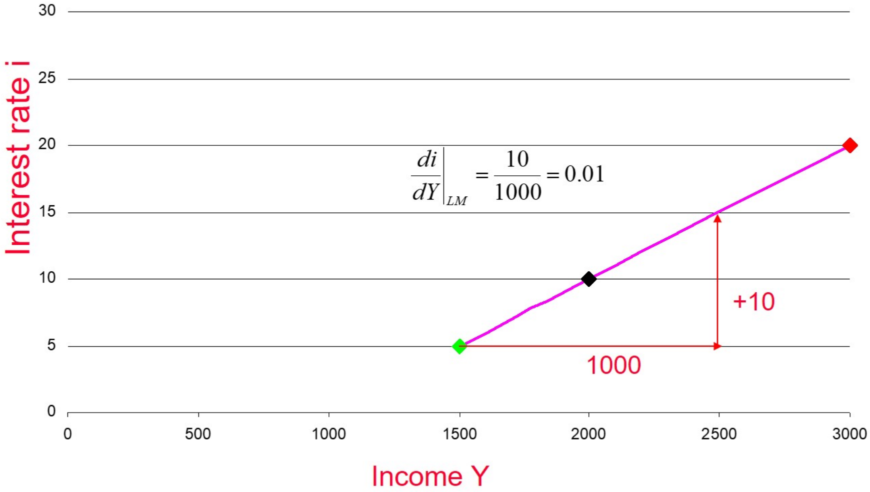
Money market equilibrium and the LM-curve ($Y = 1500$)



Money market equilibrium and the LM-curve ($Y = 3000$)



LM-curve in the interest rate-income-diagram



Slope of the LM-curve in an interest rate-income-diagram

(20)
$$\frac{M}{P} = d_0 + d_1 Y - d_2 i$$

(21)
$$\frac{1}{P} dM - \frac{M}{P^2} dP = dd_0 + d_1 dY - d_2 di$$

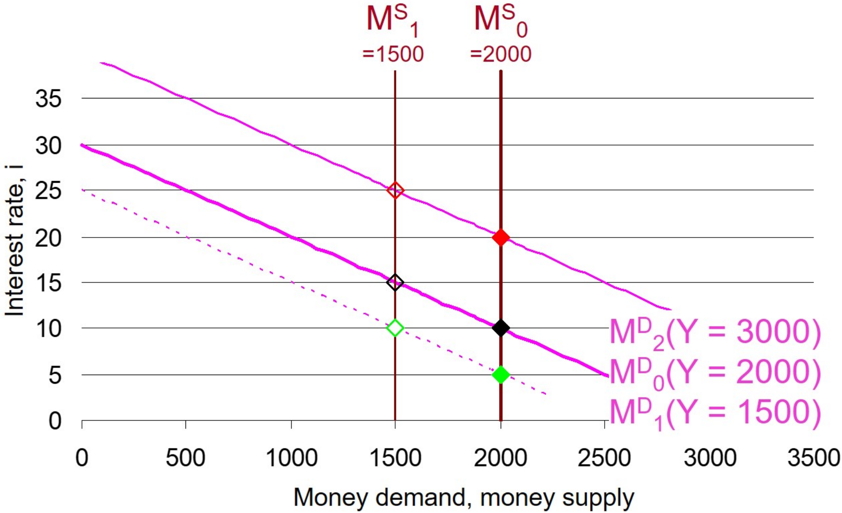
(22)
$$d_2 di = d_1 dY$$

(23)
$$\left.\frac{di}{dY}\right|_{LM} = \frac{d_1}{d_2} > 0$$

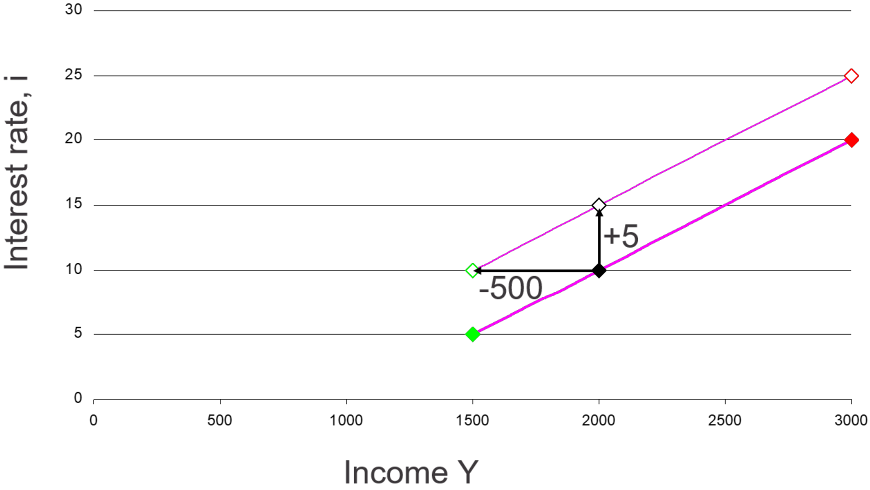
$$\left.\frac{di}{dY}\right|_{LM} = \frac{d_1}{d_2} = \frac{1}{100} = 0.01$$

d_1	$= 1$
d_2	$= 100$
d_0	$= 1000$
Y	$= 2000$
M	$= 2000$
P	$= 1$

Shift of the LM-curve: Money supply is reduced



Shift of the LM-curve



Horizontal shift of the LM-curve ($dM = -500$)

(24)
$$\frac{M}{P} = d_0 + d_1 Y - d_2 i$$

(25)
$$\frac{1}{P} dM - \frac{M}{P^2} dP = dd_0 + d_1 dY - d_2 di$$

(26)
$$d_1 dY = \frac{1}{P} dM$$

(27)
$$\left.\frac{dY}{dM}\right|_{LM} = \frac{1}{P d_1} > 0$$

Horizontal shift of the LM-curve ($dM = -500$)

$$\left.\frac{dY}{dM}\right|_{LM} = \frac{1}{P \cdot d_1} > 0$$

d_1	$= 1$
d_2	$= 100$
d_0	$= 1000$
Y	$= 2000$
M	$= 2000$
P	$= 1$

$$\frac{dY}{dM} = \frac{1}{1 \cdot 1} = 1$$

$dM = -500 \rightarrow dY = -500$ Shift to the left

Vertical shift of the LM-curve ($dM = -500$)

$$\frac{M}{P} = d_0 + d_1 Y - d_2 i \tag{28}$$

$$\frac{1}{P} dM - \frac{M}{P^2} dP = dd_0 + d_1 dY - d_2 di \tag{29}$$

$$-d_2 di = \frac{1}{P} dM \tag{30}$$

$$\left.\frac{di}{dM}\right|_{LM} = -\frac{1}{P d_2} < 0 \tag{31}$$

Vertical shift of the LM-curve ($dM = -500$)

$$\left.\frac{di}{dM}\right|_{LM} = -\frac{1}{P \cdot d_2} < 0$$

d_1	$= 1$
d_2	$= 100$
d_0	$= 1000$
Y	$= 2000$
M	$= 2000$
P	$= 1$

(32) $$\frac{di}{dM} = -\frac{1}{1 \cdot 100} = -0.01$$

$dM = -500 \rightarrow di = 5$ Shift upwards

Shift of an equilibrium curve

Shift of an equilibrium curve,

- if a variable changes,
- which is contained in the equilibrium condition,
- but is not indicated on either of the two axes.

$$\frac{M}{P} = d_0 + d_1 Y - d_2 i$$

- The LM curve shifts to the right, if
- $M \uparrow \quad P \downarrow \quad d_0 \downarrow$

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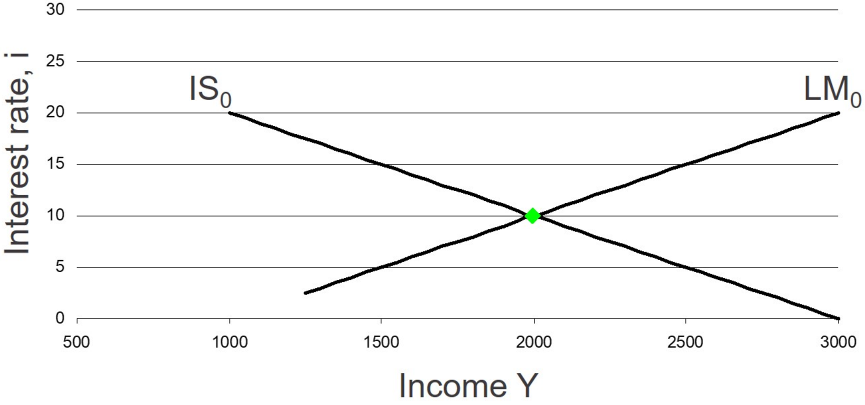
Equilibrium income and interest rate



Computation of the equilibrium *income* by inserting equations

(33) $$Y = c_0 + c_1(Y - T) + b_0 + b_1 Y - b_2 i + G$$

(34) $$\frac{M}{P} = d_0 + d_1 Y - d_2 i$$

If one solves equation (33) after the interest rate, it follows that:

(35) $$b_2 i = c_0 - c_1 T + b_0 + G - Y + c_1 Y + b_1 Y$$

(36) $$b_2 i = [c_0 - c_1 T + b_0 + G] - (1 - c_1 - b_1) Y$$

(37) $$i = \frac{[...] - (1 - c_1 - b_1) Y}{b_2} \quad \textit{with} \quad [...] = [c_0 - c_1 T + b_0 + G]$$

Computation of the equilibrium *income* by inserting equations

If one inserts equation (37) into the expression $d_1 Y - d_2 i = \frac{M}{P} - d_0$, it follows that:

$$d_1 Y - d_2 \cdot \left(\frac{[...] - (1 - c_1 - b_1) Y}{b_2} \right) = \frac{M}{P} - d_0 \tag{38}$$

$$d_1 Y + \frac{d_2 (1 - c_1 - b_1) Y}{b_2} = \frac{M}{P} - d_0 + \frac{d_2}{b_2} [...] \tag{39}$$

$$\frac{d_1 b_2 + d_2 (1 - c_1 - b_1)}{b_2} Y = \frac{M}{P} - d_0 + \frac{d_2}{b_2} [...] \tag{40}$$

Computation of the equilibrium *income* by inserting equations

(41)
$$Y = \frac{b_2[\frac{M}{P} - d_0] + d_2[...]}{d_1 b_2 + d_2(1 - c_1 - b_1)}$$

(42)
$$Y = \frac{b_2[\frac{M}{P} - d_0] + d_2[c_0 - c_1 T + b_0 + G]}{d_1 b_2 + d_2(1 - c_1 - b_1)}$$

Computation of the equilibrium *income* by inserting equations

$$Y = \frac{b_2 \cdot [\frac{M}{P} - d_0] + d_2 \cdot [c_0 - c_1 \cdot T + b_0 + G]}{d_1 \cdot b_2 + d_2 \cdot (1 - c_1 - b_1)}$$

Money Market	Goods Market
$d_1 = 1$ $d_2 = 100$	$c_1 = 0.75$ $b_1 = 0.05$ $b_2 = 20$
$d_0 = 1000$ $M = 2000$ $P = 1$	$c_0 = 400$ $b_0 = 150$ $T = 200$ $G = 200$

$$Y = \frac{20 \cdot [\frac{2000}{1} - 1000] + 100 \cdot [400 - 150 + 150 + 200]}{1 \cdot 20 + 100 \cdot (1 - 0.75 - 0.05)}$$

$$Y = \frac{20000 + 60000}{20 + 20} = 2000$$

Computation of the equilibrium *interest rate* by inserting equations

(43)
$$i = \frac{[...]}{b_2} - \frac{(1 - c_1 - b_1)}{b_2} Y$$

(44)
$$Y = \frac{b_2[\frac{M}{P} - d_0] + d_2[...]}{d_1 b_2 + d_2(1 - c_1 - b_1)}$$

(45)
$$i = \frac{[...]}{b_2} - \frac{(1 - c_1 - b_1)}{b_2} \cdot \frac{b_2[\frac{M}{P} - d_0] + d_2[...]}{[d_1 b_2 + d_2(1 - c_1 - b_1)]}$$

(46)
$$i = \frac{[...]}{b_2} - \frac{b_2(1 - c_1 - b_1)[\frac{M}{P} - d_0] + (1 - c_1 - b_1)d_2[...]}{b_2[d_1 b_2 + d_2(1 - c_1 - b_1)]}$$

Computation of the equilibrium *interest rate* by inserting equations

$$(47) \qquad i = \frac{[...]}{b_2} - \frac{b_2(1 - c_1 - b_1)[\frac{M}{P} - d_0] + (1 - c_1 - b_1)d_2[...]}{b_2[d_1 b_2 + d_2(1 - c_1 - b_1)]}$$

$$i = \frac{[...][d_1 b_2 + d_2(1 - c_1 - b_1)] - b_2(1 - c_1 - b_1)[\frac{M}{P} - d_0] - (1 - c_1 - b_1)d_2[...]}{b_2[d_1 b_2 + d_2(1 - c_1 - b_1)]}$$

$$(48) \qquad i = \frac{[...]d_1 b_2 - b_2(1 - c_1 - b_1)[\frac{M}{P} - d_0]}{b_2[d_1 b_2 + d_2(1 - c_1 - b_1)]}$$

If you cancel out b_2 from numerator and denominator, it follows that:

Computation of the equilibrium *interest rate* by inserting equations

(49)
$$i = \frac{[...]d_1 - (1 - c_1 - b_1) \cdot \left[\frac{M}{P} - d_0\right]}{d_1 b_2 + d_2(1 - c_1 - b_1)}$$

(50)
$$i = \frac{[c_0 - c_1 T + b_0 + G]d_1 - (1 - c_1 - b_1) \cdot \left[\frac{M}{P} - d_0\right]}{d_1 b_2 + d_2(1 - c_1 - b_1)}$$

Computation of the equilibrium *interest rate* by inserting equations

$$i = \frac{[c_0 - c_1 \cdot T + b_0 + G] \cdot d_1 - (1 - c_1 - b_1) \cdot \left[\frac{M}{P} - d_0\right]}{d_1 \cdot b_2 + d_2 \cdot (1 - c_1 - b_1)}$$

Money Market	Goods Market
$d_1 = 1$ $d_2 = 100$	$c_1 = 0.75$ $b_1 = 0.05$ $b_2 = 20$
$d_0 = 1000$ $M = 2000$ $P = 1$	$c_0 = 400$ $b_0 = 150$ $T = 200$ $G = 200$

$$i = \frac{[400 - 150 + 150 + 200] \cdot 1 - (1 - 0.75 - 0.05) \cdot \left[\frac{2000}{1} - 1000\right]}{1 \cdot 20 + 100 \cdot (1 - 0.75 - 0.05)}$$

$$i = \frac{[600] - (0.2) \cdot [1000]}{20 + 20} = \frac{400}{40} = 10$$

Cramer's rule: Numerical example

The following system of equations is given:

(51) $$2 \cdot x_1 - 1 = 3 \cdot x_2$$

(52) $$-6 - 4 \cdot x_2 = -5 \cdot x_1$$

If you write the two unknowns on the left side of the equation, the result is

(53) $$2 \cdot x_1 - 3 \cdot x_2 = 1$$

(54) $$5 \cdot x_1 - 4 \cdot x_2 = 6$$

Cramer's rule: Numerical example

$$2 \cdot x_1 - 3 \cdot x_2 = 1$$

$$5 \cdot x_1 - 4 \cdot x_2 = 6$$

In Matrix representation:

$$\begin{bmatrix} 2 & -3 \\ 5 & -4 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 1 \\ 6 \end{bmatrix} \tag{55}$$

System consists of the coefficient matrix $\begin{bmatrix} 2 & -3 \\ 5 & -4 \end{bmatrix}$, the vector of the two unknowns $\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$, and the solution vector $\begin{bmatrix} 1 \\ 6 \end{bmatrix}$.

Compute x_1 using Cramer's rule

$$\begin{bmatrix} 2 & -3 \\ 5 & -4 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 1 \\ 6 \end{bmatrix} \tag{56}$$

- x_1 is in the vector of the two unknowns at first position!
- Take the elements of the solution vector $\begin{bmatrix} 1 \\ 6 \end{bmatrix}$ and substitute the first column in the coefficient matrix.
- Cramer's rule: Calculate the determinant of the changed coefficient matrix and divide by the determinant of the unchanged coefficient matrix!

$$x_1 = \frac{\begin{vmatrix} 1 & -3 \\ 6 & -4 \end{vmatrix}}{\begin{vmatrix} 2 & -3 \\ 5 & -4 \end{vmatrix}} = \frac{(1)\cdot(-4)-(6)\cdot(-3)}{(2)\cdot(-4)-(5)\cdot(-3)} = \frac{-4+18}{-8+15} = \frac{14}{7} = 2 \tag{57}$$

Compute x_2 using Cramer's rule

$$\begin{bmatrix} 2 & -3 \\ 5 & -4 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 1 \\ 6 \end{bmatrix} \tag{58}$$

- x_2 is in the vector of the two unknowns at second position!
- Take the elements of the solution vector $\begin{bmatrix} 1 \\ 6 \end{bmatrix}$ and substitute the second column in the coefficient matrix.
- Cramer's rule: Calculate the determinant of the changed coefficient matrix and divide by the determinant of the unchanged coefficient matrix!

$$x_2 = \frac{\begin{vmatrix} 2 & 1 \\ 5 & 6 \end{vmatrix}}{\begin{vmatrix} 2 & -3 \\ 5 & -4 \end{vmatrix}} = \frac{2 \cdot 6 - 5 \cdot 1}{(2) \cdot (-4) - (5) \cdot (-3)} = \frac{12 - 5}{-8 + 15} = \frac{7}{7} = 1 \tag{59}$$

Cross check

Insert $x_1 = 2$ and $x_2 = 1$ into the equation system:

$$2 \cdot x_1 - 1 = 3 \cdot x_2$$

$$-6 - 4 \cdot x_2 = -5 \cdot x_1$$

$$2 \cdot 2 - 1 = 3 \cdot 1 \quad \Rightarrow \quad 3 = 3 \tag{60}$$

$$-6 - 4 \cdot 1 = -5 \cdot 2 \quad \Rightarrow \quad -10 = -10 \tag{61}$$

Check!

Determination of the equilibrium by using Cramer's rule

$$Y = c_0 + c_1(Y - T) + b_0 + b_1 Y - b_2 i + G \tag{62}$$

$$\frac{M}{P} = d_0 + d_1 Y - d_2 i \tag{63}$$

$$(1 - c_1 - b_1)Y + b_2 i = c_0 - c_1 T + b_0 + G \tag{64}$$

$$d_1 Y - d_2 i = \frac{M}{P} - d_0 \tag{65}$$

$$\begin{bmatrix} (1 - c_1 - b_1) & b_2 \\ d_1 & -d_2 \end{bmatrix} \begin{bmatrix} Y \\ i \end{bmatrix} = \begin{bmatrix} c_0 - c_1 T + b_0 + G \\ \frac{M}{P} - d_0 \end{bmatrix} \tag{66}$$

Equilibrium income level by using Cramer's rule

$$\begin{bmatrix} (1-c_1-b_1) & b_2 \\ d_1 & -d_2 \end{bmatrix} \begin{bmatrix} Y \\ i \end{bmatrix} = \begin{bmatrix} c_0 - c_1 T + b_0 + G \\ \frac{M}{P} - d_0 \end{bmatrix}$$

$$Y = \frac{\begin{vmatrix} c_0 - c_1 T + b_0 + G & b_2 \\ \frac{M}{P} - d_0 & -d_2 \end{vmatrix}}{\begin{vmatrix} (1-c_1-b_1) & b_2 \\ d_1 & -d_2 \end{vmatrix}} = \frac{[c_0 - c_1 T + b_0 + G](-d_2) - [\frac{M}{P} - d_0]b_2}{(1-c_1-b_1)(-d_2) - d_1 b_2}$$

(67) $$Y = \frac{-b_2[\frac{M}{P} - d_0] - d_2[c_0 - c_1 T + b_0 + G]}{-d_1 b_2 - d_2(1-c_1-b_1)}$$

(68) $$Y = \frac{b_2[\frac{M}{P} - d_0] + d_2[c_0 - c_1 T + b_0 + G]}{d_1 b_2 + d_2(1-c_1-b_1)}$$

(68) = (42) Slide 36

Interest rate level by using Cramer's rule

(69)
$$\begin{bmatrix} (1-c_1-b_1) & b_2 \\ d_1 & -d_2 \end{bmatrix} \begin{bmatrix} Y \\ i \end{bmatrix} = \begin{bmatrix} c_0 - c_1 T + b_0 + G \\ \frac{M}{P} - d_0 \end{bmatrix}$$

(70)
$$i = \frac{\begin{vmatrix} (1-c_1-b_1) & [c_0 - c_1 T + b_0 + G] \\ d_1 & \frac{M}{P} - d_0 \end{vmatrix}}{\begin{vmatrix} (1-c_1-b_1) & b_2 \\ d_1 & -d_2 \end{vmatrix}}$$

$$= \frac{(1-c_1-b_1)[\frac{M}{P} - d_0] - d_1[c_0 - c_1 T + b_0 + G]}{(1-c_1-b_1)(-d_2) - d_1 b_2}$$

(71)
$$i = \frac{-[c_0 - c_1 T + b_0 + G]d_1 + (1-c_1-b_1)[\frac{M}{P} - d_0]}{-d_1 b_2 - d_2(1-c_1-b_1)}$$

Interest rate level by using Cramer's rule

(72) $$i = \frac{[c_0 - c_1 T + b_0 + G]d_1 - (1 - c_1 - b_1)[\frac{M}{P} - d_0]}{d_1 b_2 + d_2(1 - c_1 - b_1)}$$

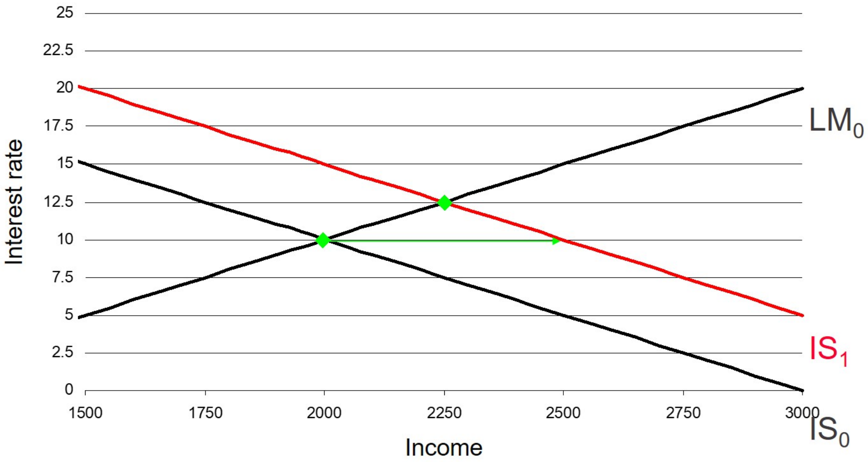
(72) = (50) Slide 40

Kapitel 5: Das IS-LM-Modell

- Learning objectives

1. Derivation of the IS-curve
   - Slope of the IS curve
   - Shift of the IS-curve
2. Derivation of the LM-curve
   - Slope of the LM-curve
   - Shift of the LM-curve
3. Interplay of IS and LM equation
   - Graphic analysis
   - Calculation of the equilibrium values
   - Cramer's rule
4. Expansionary fiscal policy
   - Transmission
   - Income multiplier
   - Interest rate multiplier
5. Expansionary monetary policy
   - Transmission
   - Income multiplier
   - Interest rate multiplier
6. Fact check

Expansionary fiscal policy ($dG = 100$)



Expansionary fiscal policy: The dynamical adjustment process

- $G\uparrow \;\Rightarrow\; Y^D\uparrow \;\Rightarrow\; Y^D > Y^S \;\Rightarrow\; Y\uparrow \;\Rightarrow\; C(Y)\uparrow \;\Rightarrow\; Y\uparrow\uparrow\uparrow\uparrow\uparrow$
- Spillover effect from the goods market to the money market:
- $M^D\uparrow \;\Rightarrow\; M^D > M^S \;\Rightarrow\; B^S\uparrow \;\Rightarrow\; B^S > B^D \;\Rightarrow\; BP\downarrow \;\Rightarrow\; i\uparrow \;\Rightarrow\; M^D\downarrow$
- Due to the rise in interest rates: Spillover effect from the money market to the goods market:
- $(-b_2 \cdot i\uparrow)\downarrow \;\Rightarrow\; Y^D\downarrow \;\Rightarrow\; Y^D < Y^S \;\Rightarrow\; Y\downarrow \;\Rightarrow\; C(Y)\downarrow \;\Rightarrow\; Y\downarrow\downarrow$
- Interest-induced crowding-out slows down the increase in income.

Income multiplier of a fiscal policy

(73) $$\begin{bmatrix}(1-c_1-b_1) & b_2\\ d_1 & -d_2\end{bmatrix}\begin{bmatrix}Y\\ i\end{bmatrix}=\begin{bmatrix}c_0-c_1T+b_0+G\\ \frac{M}{P}-d_0\end{bmatrix}$$

(74) $$\begin{bmatrix}(1-c_1-b_1) & b_2\\ d_1 & -d_2\end{bmatrix}\begin{bmatrix}dY\\ di\end{bmatrix}=\begin{bmatrix}dc_0-c_1dT+db_0+dG\\ \frac{dM}{P}-\frac{M}{P^2}dP-dd_0\end{bmatrix}$$

(75) $$\begin{bmatrix}(1-c_1-b_1) & b_2\\ d_1 & -d_2\end{bmatrix}\begin{bmatrix}dY\\ di\end{bmatrix}=\begin{bmatrix}dG\\ 0\end{bmatrix}$$

(76) $$dY=\frac{\begin{vmatrix}dG & b_2\\ 0 & -d_2\end{vmatrix}}{\begin{vmatrix}(1-c_1-b_1) & b_2\\ d_1 & -d_2\end{vmatrix}}=\frac{dG(-d_2)-0\cdot b_2}{(1-c_1-b_1)(-d_2)-d_1b_2}$$

Income multiplier of a fiscal policy

$$(77) \quad dY = \frac{dG(-d_2) - 0 \cdot b_2}{(1 - c_1 - b_1)(-d_2) - d_1 b_2} = \frac{-d_2}{-d_2(1 - c_1 - b_1) - d_1 b_2} dG$$

$$(78) \quad \frac{dY}{dG} = \frac{d_2}{d_2(1 - c_1 - b_1) + d_1 b_2}$$

If the right side of the equation is multiplied by $\frac{\frac{1}{d_2}}{\frac{1}{d_2}}$, it follows:

$$(79) \quad \frac{dY}{dG} = \frac{1}{(1 - c_1 - b_1) + \frac{d_1 b_2}{d_2}} > 0$$

Income multiplier of a fiscal policy

$$\frac{dY}{dG} = \frac{1}{(1 - c_1 - b_1) + \frac{d_1 b_2}{d_2}} > 0$$

Money Market	Goods Market
$d_1 = 1$ $d_2 = 100$	$c_1 = 0.75$ $b_1 = 0.05$ $b_2 = 20$
$d_0 = 1000$ $M = 2000$ $P = 1$	$c_0 = 400$ $b_0 = 150$ $T = 200$ $G = 200$

(80) $$\frac{dY}{dG} = \frac{1}{(1 - 0.75 - 0.05) + \frac{1 \cdot 20}{100}} = \frac{1}{0.2 + 0.2} = 2.5$$

- Here: $dG = 100 \rightarrow dY = 2.5 \cdot 100 = 250$

Interest rate multiplier of a fiscal policy

(81)
$$\begin{bmatrix} (1-c_1-b_1) & b_2 \\ d_1 & -d_2 \end{bmatrix} \begin{bmatrix} dY \\ di \end{bmatrix} = \begin{bmatrix} dG \\ 0 \end{bmatrix}$$

(82)
$$di = \frac{\begin{vmatrix} (1-c_1-b_1) & dG \\ d_1 & 0 \end{vmatrix}}{\begin{vmatrix} (1-c_1-b_1) & b_2 \\ d_1 & -d_2 \end{vmatrix}} = \frac{(1-c_1-b_1)\cdot 0 - d_1 dG}{(1-c_1-b_1)(-d_2) - d_1 b_2}$$

Interest rate multiplier of a fiscal policy

(83) $$di = \frac{(1 - c_1 - b_1) \cdot 0 - d_1 dG}{(1 - c_1 - b_1)(-d_2) - d_1 b_2} = \frac{-d_1 dG}{-d_2(1 - c_1 - b_1) - d_1 b_2}$$

(84) $$\frac{di}{dG} = \frac{d_1}{d_2(1 - c_1 - b_1) + d_1 b_2} > 0$$

Interest rate multiplier of a fiscal policy

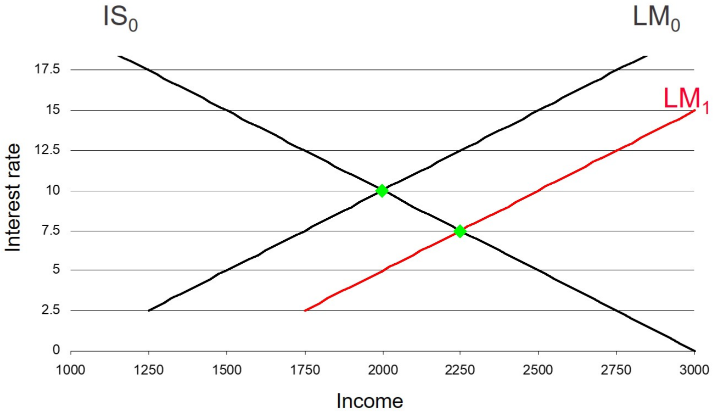
$$\frac{di}{dG} = \frac{d_1}{d_2 \cdot (1 - c_1 - b_1) + d_1 \cdot b_2} > 0$$

Money Market	Goods Market
$d_1 = 1$ $d_2 = 100$	$c_1 = 0.75$ $b_1 = 0.05$ $b_2 = 20$
$d_0 = 1000$ $M = 2000$ $P = 1$	$c_0 = 400$ $b_0 = 150$ $T = 200$ $G = 200$

(85) $$\frac{di}{dG} = \frac{1}{100 \cdot (1 - 0.75 - 0.05) + 1 \cdot 20} = \frac{1}{20 + 20} = 0.025$$

- Here: $dG = 100 \rightarrow di = 0.025 \cdot 100 = 2.5$

Expansionary monetary policy ($dM = 500$)



- Income: $+ 250$
- Interest rate: $- 2.5$

Expansionary monetary policy: The dynamical adjustment process

- $M^S \uparrow \;\Rightarrow\; M^S > M^D \;\Rightarrow\; B^D \uparrow \;\Rightarrow\; B^D > B^S \;\Rightarrow\; BP \uparrow \;\Rightarrow\; i \downarrow \;\Rightarrow\; M^D \uparrow$
- Due to the reduction in interest rates: spillover effect from the money market to the goods market:
- $(-b_2 \cdot i \downarrow) \uparrow \;\Rightarrow\; Y^D \uparrow \;\Rightarrow\; Y^D > Y^S \;\Rightarrow\; Y \uparrow \;\Rightarrow\; C(Y) \uparrow \;\Rightarrow\; Y \uparrow\uparrow\uparrow$
- Due to income growth: spillover effect from the goods market to the money market:
- $M^D \uparrow$

Income multiplier of a monetary policy

(86) $$\begin{bmatrix}(1-c_1-b_1) & b_2\\ d_1 & -d_2\end{bmatrix}\begin{bmatrix}dY\\ di\end{bmatrix}=\begin{bmatrix}dc_0-c_1dT+db_0+dG\\ \frac{dM}{P}-\frac{M}{P^2}dP-dd_0\end{bmatrix}$$

(87) $$\begin{bmatrix}(1-c_1-b_1) & b_2\\ d_1 & -d_2\end{bmatrix}\begin{bmatrix}dY\\ di\end{bmatrix}=\begin{bmatrix}0\\ \frac{1}{P}dM\end{bmatrix}$$

(88) $$dY=\frac{\begin{vmatrix}0 & b_2\\ \frac{1}{P}dM & -d_2\end{vmatrix}}{\begin{vmatrix}(1-c_1-b_1) & b_2\\ d_1 & -d_2\end{vmatrix}}=\frac{0\cdot(-d_2)-\frac{1}{P}dMb_2}{(1-c_1-b_1)(-d_2)-d_1b_2}$$

Income multiplier of a monetary policy

(89) $$dY = \frac{0 \cdot (-d_2) - \frac{1}{P} dM b_2}{(1 - c_1 - b_1)(-d_2) - d_1 b_2} = \frac{-\frac{1}{P} dM b_2}{-d_2(1 - c_1 - b_1) - d_1 b_2}$$

(90) $$\frac{dY}{dM} = \frac{\frac{1}{P} b_2}{d_2(1 - c_1 - b_1) + d_1 b_2}$$

Multiplying the right side of the equation by $\frac{\frac{1}{b_2}}{\frac{1}{b_2}}$ follows:

(91) $$\frac{dY}{dM} = \frac{\frac{1}{P}}{(1 - c_1 - b_1) \cdot \frac{d_2}{b_2} + d_1} > 0$$

Income multiplier of a monetary policy

$$\frac{dY}{dM} = \frac{\frac{1}{P}}{(1 - c_1 - b_1) \cdot \frac{d_2}{b_2} + d_1} > 0$$

Money Market	Goods Market
$d_1 = 1$ $d_2 = 100$	$c_1 = 0.75$ $b_1 = 0.05$ $b_2 = 20$
$d_0 = 1000$ $M = 2000$ $P = 1$	$c_0 = 400$ $b_0 = 150$ $T = 200$ $G = 200$

$$\frac{dY}{dM} = \frac{\frac{1}{1}}{(1 - 0.75 - 0.05) \cdot \frac{100}{20} + 1} = \frac{1}{0.2 \cdot 5 + 1} = 0.5 \tag{92}$$

- Here: $dM = 500 \rightarrow dY = 0.5 \cdot 500 = 250$

Interest rate multiplier of a monetary policy

(93)
$$\begin{bmatrix} (1-c_1-b_1) & b_2 \\ d_1 & -d_2 \end{bmatrix} \begin{bmatrix} dY \\ di \end{bmatrix} = \begin{bmatrix} 0 \\ \frac{1}{P}dM \end{bmatrix}$$

(94)
$$di = \frac{\begin{vmatrix} (1-c_1-b_1) & 0 \\ d_1 & \frac{1}{P}dM \end{vmatrix}}{\begin{vmatrix} (1-c_1-b_1) & b_2 \\ d_1 & -d_2 \end{vmatrix}} = \frac{(1-c_1-b_1)\frac{1}{P}dM - d_1 0}{(1-c_1-b_1)(-d_2) - d_1 b_2}$$

Interest rate multiplier of a monetary policy

$$(95)\qquad di = \frac{(1 - c_1 - b_1)\frac{1}{P}dM - d_1 0}{(1 - c_1 - b_1)(-d_2) - d_1 b_2} = \frac{(1 - c_1 - b_1)\frac{1}{P}dM}{-d_2(1 - c_1 - b_1) - d_1 b_2}$$

$$(96)\qquad \frac{di}{dM} = -\frac{(1 - c_1 - b_1)\frac{1}{P}}{d_2(1 - c_1 - b_1) + d_1 b_2} < 0$$

Interest rate multiplier of a monetary policy

$$\frac{di}{dM} = -\frac{(1 - c_1 - b_1) \cdot \frac{1}{P}}{d_2 \cdot (1 - c_1 - b_1) + d_1 \cdot b_2} < 0$$

Geldmarkt	Gütermarkt
$d_1 = 1$ $d_2 = 100$	$c_1 = 0.75$ $b_1 = 0.05$ $b_2 = 20$
$d_0 = 1000$ $M = 2000$ $P = 1$	$c_0 = 400$ $b_0 = 150$ $T = 200$ $G = 200$

$$\frac{di}{dM} = -\frac{(1 - 0.75 - 0.05) \cdot \frac{1}{1}}{100 \cdot (1 - 0.75 - 0.05) + 1 \cdot 20} = -\frac{0.2}{40} = -0.005 < 0 \quad (97)$$

- Here: $dM = 500 \rightarrow di = -0.005 \cdot 500 = -2.5$

5.5 How does the IS-LM-model fits the facts?

Econometric simulation study: The Empirical Effects of an Increase in the Federal Funds Rate

In the short term an increase of the interest rate reduces output and increases unemployment. Goods prices do not react on impact but only over time...