Chapter 3: National Income: Where it comes from and where it goes

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Chapter 3: National Income

Learning Goals of chapter 3

After this chapter, you could

a) be able to compute MPL and MPK,
b) know which variables are exogenous and endogenous for a firm,
c) know what kind of factors influence distribution of GDP,
d) know the interest rate balances supply and demand for goods.

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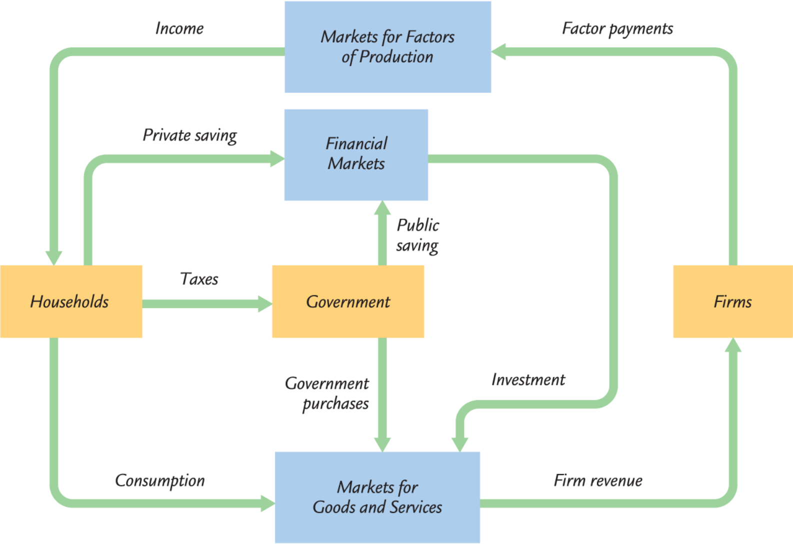
The circular flow



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The circular flow: Important aspects

Factor payments

- Wage rate (W): Compensation for the factor *Labor* (L).
- Rent (R): Compensation for the factor *Capital* (K).

Public savings $(T - G)$: Can be positive or negative!

- Positive ($T > G$): Government budget surplus.
- Negative $(T < G)$: Government budget deficit.

Factors of production

- Factors of production within the economy are fixed.
- A bar above a variable ('overbar') indicates this.

$$K = \bar{K} \tag{1}$$

$$L = \bar{L} \tag{2}$$

Classical model

- No unemployment (Unemployment is explained in Chapter 7).
- Factors of production are fully utilized!

Production function

$$Y = F(K, L) \tag{3}$$

F: Function!

- Production function has a characteristic called *"constant return to scale"*.
- *"When inputs double, output doubles."*
- Output corresponds in *a proportional way* to variations in inputs.

$$z \cdot Y = F(z \cdot K, z \cdot L) \tag{4}$$

Constant returns to scale: A numerical example

The production function in Denmark is given by:

$$Y = A \cdot K^{0.5} \cdot L^{0.5} \tag{5}$$

- The variable A symbolizes the technology. We assume $A = 1$
- Let's assume that $K = 4$ and also $L = 4$

Denmark: $K = 4$ and $L = 4$








Output in Denmark

$$Y = A \cdot K^{0.5} \cdot L^{0.5}$$

$$Y = \sqrt{K} \cdot \sqrt{L} \tag{6}$$

Output:

$$Y = \sqrt{4} \cdot \sqrt{4} = 2 \cdot 2 = 4 \tag{7}$$

Sweden compared to Denmark

Assumption:

- Sweden is 125 % larger than Denmark.
- $z = 2.25$

$$L_{Swe} = z \cdot L_{Den} = 2.25 \cdot 4 = 9 \tag{8}$$

$$K_{Swe} = z \cdot K_{Den} = 2.25 \cdot 4 = 9 \tag{9}$$

Sweden: $K = 9$ and $L = 9$

Output in Sweden

$$Y = \sqrt{K} \cdot \sqrt{L} \tag{10}$$

Output:

$$Y = \sqrt{9} \cdot \sqrt{9} = 3 \cdot 3 = 9 \tag{11}$$

$$Y_{Swe} = z \cdot Y_{Den} = 2.25 \cdot 4 = 9 \tag{12}$$

$$z \cdot Y = F(z \cdot K, z \cdot L) \tag{13}$$

Components of the model & assumptions

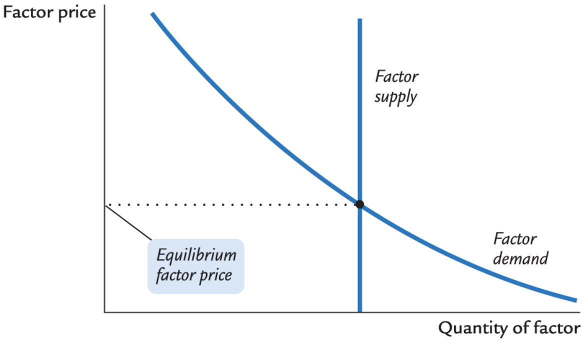
Classical idea (eighteenth-century)

- Prices balance demand and supply

Neoclassical theory of distribution (nineteenth-century)

- Demand for each factor of production depends on the marginal productivity of that factor.

Factor price



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Competitive firm

- A competitive firm is relatively small.
- It has no market power to influence prices.
- Prices are given (=prices are exogenous TO THE FIRM)
- What kind of prices?
- Price for the output as well as prices for the inputs!
  - The goods price (P) is given.
  - The wage rate (W) is given.
  - The rent (R) is given.

Competitive firm

- What is under control of the firm?
- What kind of decisions have to be made?
  - Choose the optimal amount of labor (L).
  - Choose the optimal amount of capital (K).

Objective of the firm: Maximize profits

- We need a profit function!
- Profit = Revenues minus Cost
    - Revenues: Price of the good * quantity of output
    - Cost: Cost for the factors of production

$$Profit = P \cdot Y - W \cdot L - R \cdot K \tag{14}$$

$$Profit = P \cdot F(K, L) - W \cdot L - R \cdot K \tag{15}$$

Demand for factors: Depends on MPL

- MPL = Marginal productivity of labor...
- ... is defied as the extra amount of output from one extra unit of labor, holding the amount of capital constant.

$$MPL = F(K, L+1) - F(K, L) \tag{16}$$

MPL – Numerical example: $Y = \sqrt{K} \cdot \sqrt{L}$

- Initially: $K = 100$ and $L = 100$
- $Y = \sqrt{K} \cdot \sqrt{L} = \sqrt{100} \cdot \sqrt{100} = 10 \cdot 10 = 100$

- Afterwards: $K = 100$ and $L = 101$
- $Y = \sqrt{K} \cdot \sqrt{L} = \sqrt{100} \cdot \sqrt{101} \approx 10 \cdot 10.05 \approx 100.5$

$$MPL = F(K, L+1) - F(K, L) = 100.5 - 100 = 0.5 \tag{17}$$

MPL – First derivative of production function

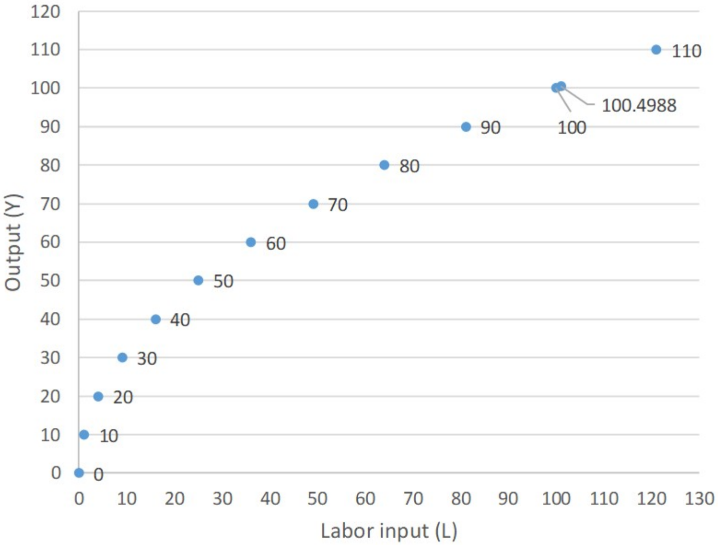
$$MPL = \frac{\partial F}{\partial L} \tag{18}$$

The production function is given by: $F = K^{0.5} \cdot L^{0.5}$

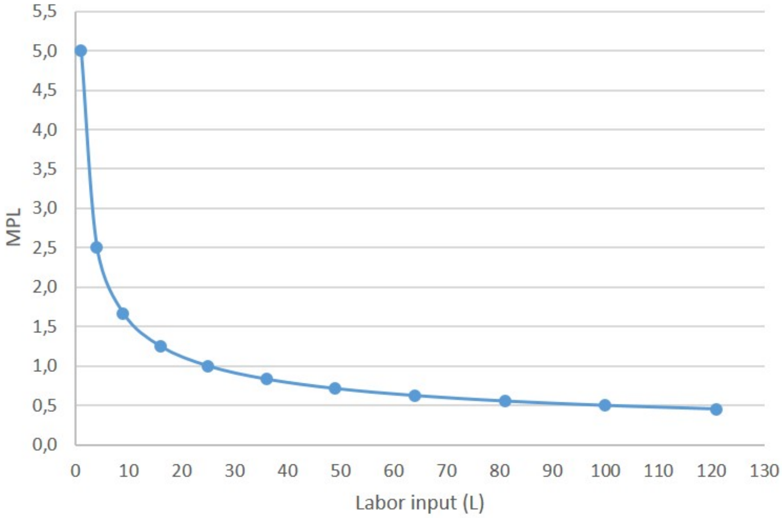
$$MPL = \frac{\partial F}{\partial L} = 0.5 \cdot K^{0.5} \cdot L^{0.5-1} = 0.5 \cdot K^{0.5} \cdot L^{-0.5} \tag{19}$$

$$MPL = 0.5 \cdot \frac{K^{0.5}}{L^{0.5}} = 0.5 \cdot \frac{100^{0.5}}{100^{0.5}} = 0.5 \tag{20}$$

Production function



Marginal Product of Labor (*MPL*) measured in output units



Profit maximum

Factors of production

Profit maximum?

- After graduation, you take over the responsibility for the *"Hasmark Grill & Minigolf"*
- On your first day of work, you are checking, whether the business is operating at its profit maximum.
- Production factors: $K = 100$ & $L = 1$.
- An ice cream (Hasmark Waffel) can be sold at $P = 30$ DKK and the labor unions have fixed the nominal wage at $W = 90$ DKK.
- You can still remember that the production function in each and every business is given by:

$$F = K^{0.5} \cdot L^{0.5} \tag{21}$$

Assumptions

- Capital is fixed and can not be adjusted in the short run.
- Only labor is variable...
- What do you do?

Profit maximum?

1. Calculate MPL!

$$\frac{\partial F}{\partial L} = 0.5 \cdot \frac{K^{0.5}}{L^{0.5}} = 0.5 \cdot \frac{100^{0.5}}{1^{0.5}} = 0.5 \cdot \frac{10}{1} = 5 \qquad (22)$$

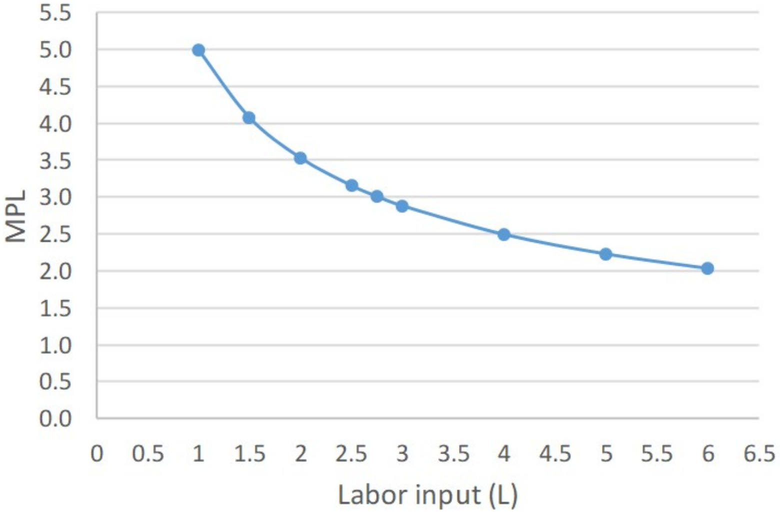
- One additional worker could create 5 additional ice creams (per hour).
- Ice cream can be sold at $P = 30$.
- "Marginal revenues" of one additional worker is $P \cdot MPL = 5 \cdot 30 = 150$ DKK.
- The marginal cost of one additional worker is $W = 90$ DKK.
- Profit would increase by 60 DKK!
- Conclusion: Hire more workers!

Profit maximum?

- As long as $P \cdot MPL > W \Rightarrow$ Hire more workers.
- Labor demand of Hasmark Grill does not affect the Denmarks' wage rate W.
- Additional output does not affect the price of ice cream P.
- But additional labor input affects MPL!

$$P \cdot MPL \downarrow = W \tag{23}$$

Marginal Product of Labor (*MPL*)



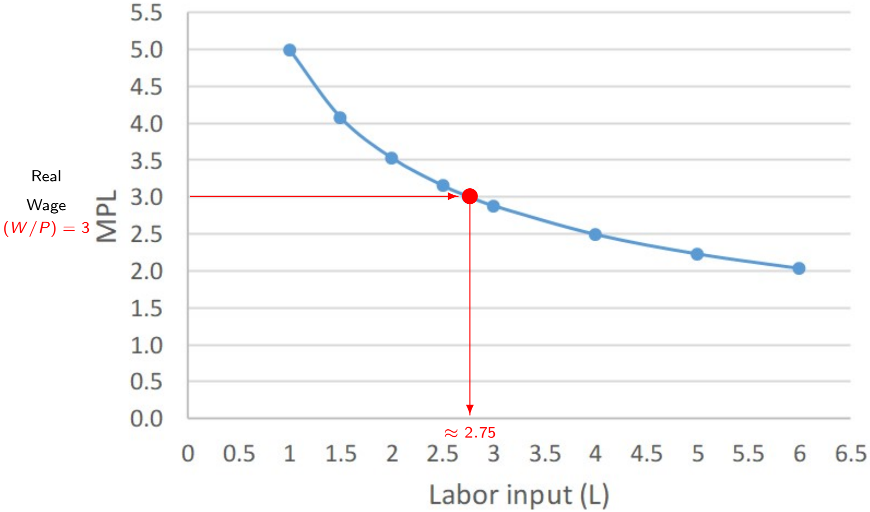
Profit maximum

$$P \cdot MPL = W$$

Relationship between MPL and real wage:

$$MPL = \frac{W}{P} = \frac{90}{30} = 3 \tag{24}$$

Marginal Product of Labor (*MPL*)



Recap Additional slide

- MPL= Marginal Productivity of Labor
- How much does output increase in case that labor increases by one unit?
- $Y \uparrow = ?$ if $dL = +1$?
- Profit maximization condition:

$$MPL \cdot P = W \tag{25}$$

$$MPL = \frac{W}{P} \tag{26}$$

- In the optimum: MPL is equal to the real wage.

Objective of the firm: Maximize profits Additional slide

- Profit = Revenues minus Cost

$$Profit_{DKK} = P \cdot Y - W \cdot L - R \cdot K \tag{27}$$

$$Profit_{DKK} = P \cdot F(K, L) - W \cdot L - R \cdot K \tag{28}$$

Divide by the price (P):

$$\frac{Profit_{DKK}}{P} = F(K, L) - \frac{W}{P} \cdot L - \frac{R}{P} \cdot K \tag{29}$$

Economic profit = $\frac{Profit_{DKK}}{P}$:

$$\text{Economic profit} = Y - MPL \cdot L - MPK \cdot K \tag{30}$$

Division of National Income

$$\text{Economic profit} = Y - (MPL \cdot L) - (MPK \cdot K) \tag{25}$$

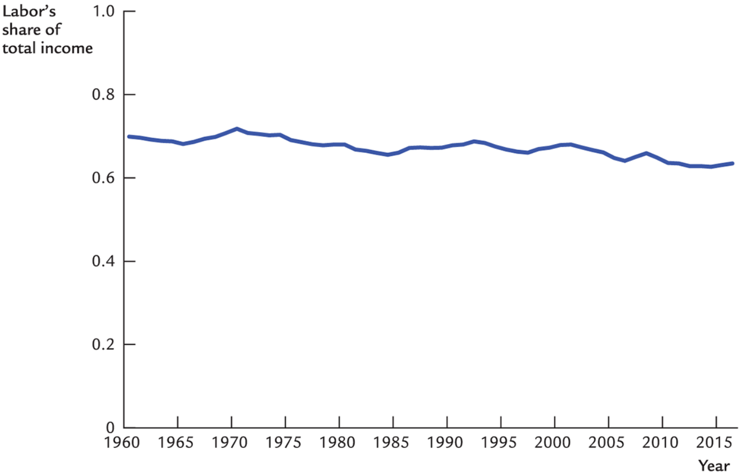
$$\text{Economic profit} + (MPL \cdot L) + (MPK \cdot K) = Y \tag{26}$$

$$Y = \text{Economic profit} + (MPL \cdot L) + (MPK \cdot K) \tag{27}$$

Market structure of perfect competition (long run equilibrium): Economic profit is zero.

$$Y = (MPL \cdot L) + (MPK \cdot K) \tag{28}$$

The output is divided between the payments to capital and the payments to labor, depending on their marginal productivities.



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Cobb-Douglas production function

$$F(K, L) = A \cdot K^{\alpha} \cdot L^{1-\alpha} \tag{29}$$

$$MPL = \frac{\partial F}{\partial L} = (1-\alpha) \cdot A \cdot K^{\alpha} \cdot L^{1-\alpha-1} = (1-\alpha) \cdot A \cdot K^{\alpha} \cdot L^{-\alpha} \tag{30}$$

$$MPK = \frac{\partial F}{\partial K} = \alpha \cdot A \cdot K^{\alpha-1} \cdot L^{1-\alpha} \tag{31}$$

Cobb-Douglas production function

$$MPL = (1-\alpha) \cdot A \cdot K^{\alpha} \cdot L^{-\alpha} \tag{32}$$

$$MPL = (1-\alpha) \cdot A \cdot K^{\alpha} \cdot L^{-\alpha} \cdot \frac{L}{L} \tag{33}$$

$$MPL = (1-\alpha) \cdot \frac{A \cdot K^{\alpha} \cdot L \cdot L^{-\alpha}}{L} = (1-\alpha) \cdot \frac{A \cdot K^{\alpha} \cdot L^{1-\alpha}}{L} \tag{34}$$

$$MPL = (1-\alpha) \cdot \frac{Y}{L} \tag{35}$$

$$MPL \cdot L = (1-\alpha) \cdot Y \tag{36}$$

Same procedure for capital

$$MPL \cdot L = (1 - \alpha) \cdot Y \tag{37}$$

$$MPK \cdot K = \alpha \cdot Y \tag{38}$$

Dividing (37) by (38) yields:

$$\frac{MPL \cdot L}{MPK \cdot K} = \frac{(1 - \alpha) \cdot Y}{\alpha \cdot Y} \tag{39}$$

$$\frac{MPL \cdot L}{MPK \cdot K} = \frac{1 - \alpha}{\alpha} \tag{40}$$

Labor shares of total income

$$\frac{MPL \cdot L}{MPK \cdot K} = \frac{1-\alpha}{\alpha}$$

- The ratio of labor income to capital income is a constant ($\frac{1-\alpha}{\alpha}$).
- The factor shares depend only on the parameter α,
  - not on the amounts of capital (K),
  - not on the amounts of labor (L),
  - not on the state of technology (as measured by the variable A).

What does it imply for our numerical example?

$$\frac{MPL \cdot L}{MPK \cdot K} = \frac{1-\alpha}{\alpha}$$

- The production function was given by: $F = K^{0.5} \cdot L^{0.5}$
- $\alpha = 0.5$

$$\frac{MPL \cdot L}{MPK \cdot K} = \frac{1-\alpha}{\alpha} = \frac{1-0.5}{0.5} = 1$$

What does it imply for our numerical example?

$$\frac{MPL \cdot L}{MPK \cdot K} = \frac{1-\alpha}{\alpha} = \frac{1-0.5}{0.5} = 1$$

Let's check!

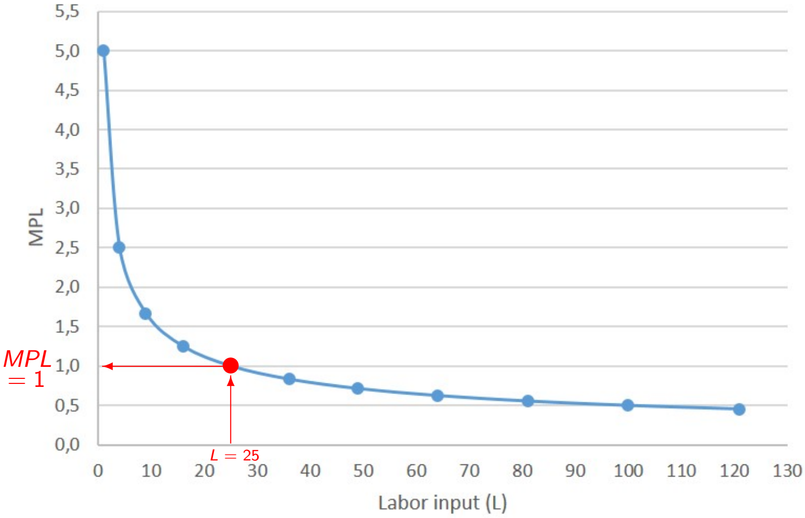
- Let's assume: $L = 25$ and $K = 100$

a) What is the equilibrium value of the real wage (=the marginal productivity of labor) ($W/P = MPL$)?

b) What is the equilibrium value of the real rent (=the marginal productivity of capital) ($R/P = MPC$)?

c) How large is the output (Y) and how is it split up?

Part a) MPL when $\bar{K} = 100$

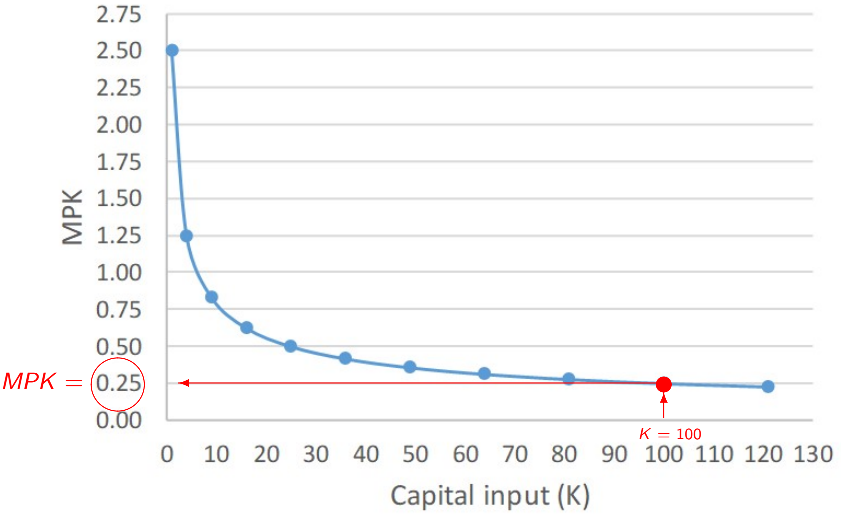


Part b) Marginal productivity of capital (MPK)

- The production function was given by: $F = K^{0.5} \cdot L^{0.5}$

$$MPK = \frac{\partial F}{\partial K} = 0.5 \cdot K^{0.5-1} \cdot L^{0.5} = 0.5 \cdot \frac{L^{0.5}}{K^{0.5}} \tag{41}$$

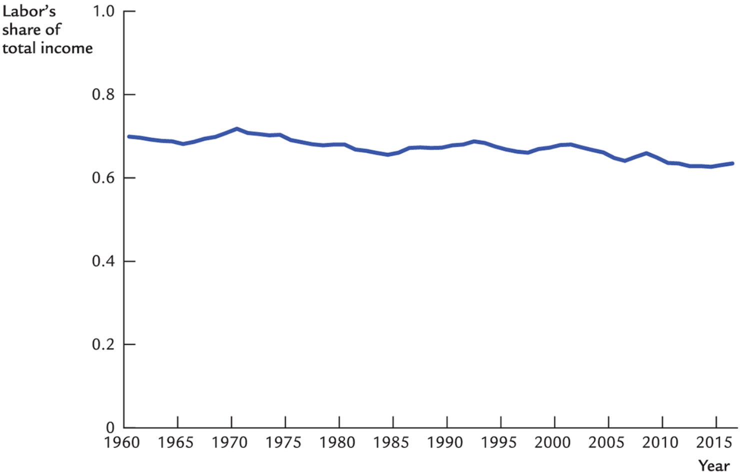
Part b) MPK when $\bar{L} = 25$



Part c) How large is the output (Y) and how is it split up?

- The production function was given by: $Y = K^{0.5} \cdot L^{0.5}$
- Output: $Y = 100^{0.5} \cdot 25^{0.5} = 10 \cdot 5 = 50$

$$\frac{MPL \cdot L}{MPK \cdot K} = \frac{1.0 \cdot 25}{0.25 \cdot 100} = \frac{25}{25} = 1 \tag{42}$$



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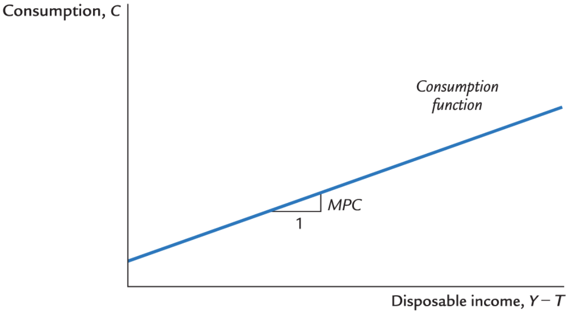
Composition of GDP

$$Y = C + I + G + NX \tag{43}$$

- Closed economy: Exports = 0 & Imports = 0
- Net exports zero: $NX = 0$

$$Y = C + I + G \tag{44}$$

Consumption



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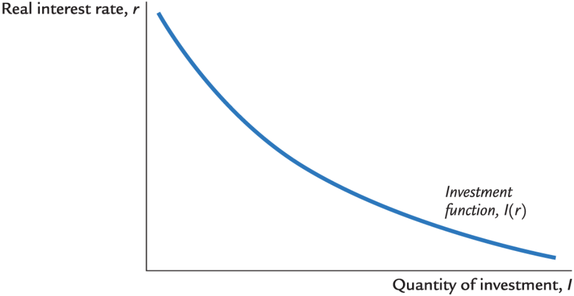
Consumption

$$C = C(Y - T) \tag{45}$$

$$C = c_0 + c_1 \cdot (Y - T) \tag{46}$$

- c_0: Autonomous component of consumption.
- $1 > c_1 > 0$: Marginal propensity to consume (MPC).

Investment



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Investment

$$I = I(r) \tag{47}$$

- Numerical example No. 10 (p. 77):

$$I = 1200 - 100 \cdot r \tag{48}$$

$$I = b_0 - b_2 \cdot r \tag{49}$$

- b_0: Autonomous component of investment.
- b_2: Interest rate reactivity of investment.

Real versus nominal interest rate

- You have $x_t = 100$ EUR
- Nominal interest rate (i = 10%)
- Price of a chocolate bar in period t: $P_t = 1.00$ EUR/Choc
- How many choc bars can you buy in the beginning?

$$\frac{x_t}{P_t} = \frac{100EUR}{1EUR/choc} = 100choc \tag{50}$$

One year later... Scenario A

- No inflation $\Rightarrow$ Good price is constant.
- Price of a chocolate bar in period $t+1$: $P_{t+1} = 1.00$ EUR/Choc
- Financial wealth has increase from $x_t = 100$ EUR to $x_{t+1} = 110$ EUR, due to the interest payment.

- How many choc bars can you buy in period $t+1$?

$$\frac{x_{t+1}}{P_{t+1}} = \frac{110EUR}{1EUR/choc} = 110choc \tag{51}$$

- The amount of choc increased by 10 %.
- The real interest rate is $r = 10$ %.

One year later... Scenario B

- Inflation rate $\pi = 10\%$.
- Price of a chocolate bar in period $t+1$: $P_{t+1} = 1.10$ EUR/Choc
- Financial wealth has increase from $x_t = 100$ EUR to $x_{t+1} = 110$ EUR, due to the interest payment.

- How many choc bars can you buy in period $t+1$?

$$\frac{x_{t+1}}{P_{t+1}} = \frac{110EUR}{1.1EUR/choc} = 100choc \tag{52}$$

- The amount of choc is constant.
- The real interest rate is $r = 0\%$.

One year later... Scenario C

- Inflation rate $\pi = 5\%$.
- Price of a chocolate bar in period $t+1$: $P_{t+1} = 1.05$ EUR/Choc
- Financial wealth has increase from $x_t = 100$ EUR to $x_{t+1} = 110$ EUR, due to the interest payment.

- How many choc bars can you buy in period $t+1$?

$$\frac{x_{t+1}}{P_{t+1}} = \frac{110 EUR}{1.05 EUR/choc} \approx 105 choc \tag{53}$$

- The amount of choc is up by 5 units.
- The real interest rate is $r = 5$ %.

Relationship between nominal and real interest rate

$$r = i - \pi \tag{54}$$

Government purchases

- $G = \bar{G}$
- $T = \bar{T}$

VERY important remarks: Setting

- The model can be classified as a ”*classical*” model.
- The interest rate adjusts and equilibrates supply and demand.
- Two ways of thinking:
  - Interest rate affects demand for goods and services (not the supply!).
  - Interest rate affects the demand (and maybe also the supply) of loanable funds.

Demand and supply: Good market

Demand

$$Y = C(Y - \bar{T}) + I(r) + \bar{G} \tag{55}$$

Supply

$$Y = F(\bar{K}, \bar{L}) \tag{56}$$

Goods supply is exogenous:

$$\bar{Y} = F(\bar{K}, \bar{L}) \tag{57}$$

Supply = Demand

$$\bar{Y} = C(\bar{Y} - \bar{T}) + I(r) + \bar{G} \tag{58}$$

Demand and supply: Good market

$$\bar{Y} = C(\bar{Y} - \bar{T}) + I(r) + \bar{G}$$

- The interest rate (r) is the only variable not already determined in the last equation.
- How does the interest rate adjusts?
- The best way to understand is to consider *financial markets* in this story.

Saving & investment

$$Y = C + I(r) + G \quad \Big| - C - G \tag{59}$$

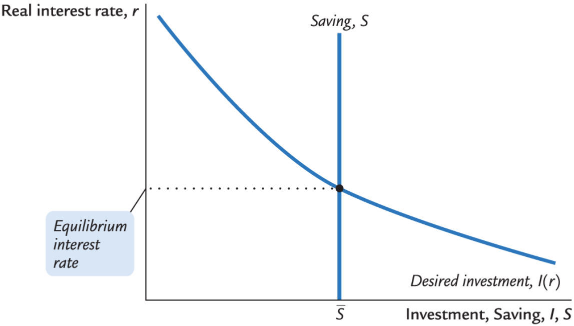
$$Y - C - G = I(r) \quad \Big| - T + T \text{ 'only' on the left hand side} \tag{60}$$

$$Y - T - C + (T - G) = I(r) \tag{61}$$

- Private saving: $Y - T - C$
- Public saving: $T - G$
- National saving (S) = Private saving + Public saving

$$\bar{S} = I(r) \tag{62}$$

Equilibrium in financial markets



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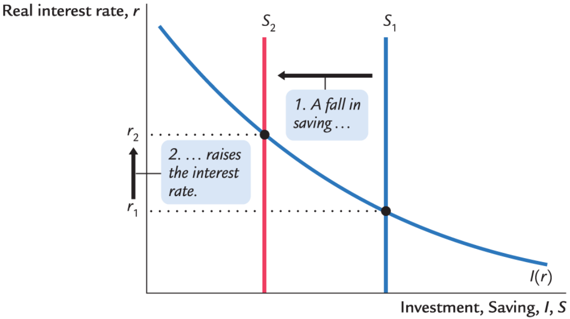
Increase in government spending crowds out private investment



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